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Iterative Temperature Calculation Method for Rectangular Sandwich Panel Fins

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An iterative calculation method is developed for the steady-state temperature distribution in a two-dimensional rectangular sandwich panel fin heated within a rectangular footprint region, and dissipating energy to its environments as linearized radiation. This method enables the temperature of each panel skin to be individually calculated by controlling mean skin temperatures under an energy balance constraint. The formulations derived approximate the typical satellite application in which a heat-generating electronic component is mounted on a equipment panel, such as an aluminum honeycomb sandwich panel. Comparison of numerical results obtained from the proposed method, and lumped nodal network analysis, shows that the proposed method will be useful for the evaluation of sandwich panel radiation fins in trade studies where geometrical configuration, heat loads, thermal properties, and environmental parameters change frequently.

Nomenclature

A = surface area, m^2
b = width in Y direction, m
F = configuration factor
$\mathcal{F}$ = radiation interchange factor
H = heat dissipation coefficient for linearized radiation, W/m^2K
h = heat transfer rate between skins of sandwich panel, W/m^2K
k = thermal conductivity, W/mK
l = length in X direction, m
q = heat flux, W/m^2
T = temperature, K
t_1, t_2 = thickness of inner and outer skin, m
ε = emissivity
σ = Stefan-Boltzmann constant, 5.67×10^{-8} W/m^2K^4

Subscripts
a = electronic component side environment
f = footprint
i = inner skin, or footprint side surface
o = outer skin surface
s = external space environment

Introduction

PASSIVE thermal control techniques are commonly used to control the temperatures of satellite electronic equipment. Electronic components requiring high-heat dissipation are usually mounted on high-thermal conductivity plate elements. The plate elements, often referred to as heat-sink plates or thermal doublers, are bonded to honeycomb sandwich equipment panels. The panels transfer heat energy from the satellite into space. A sandwich panel combined with such a plate element can be treated as a radiation fin. The greater the heat flux on a component footprint, the higher the capacity of the fin should be to maintain the component's temperature within the required range.

Temperature calculations of satellites are usually performed utilizing the lumped nodal network analysis method. However, if fin optimization is carried out, using the results obtained from a nodal thermal model constructed for a satellite or an equipment panel, it is necessary to establish a nodal model that has a great number of nodes that include the maximum temperature and/or the specified monitoring points. Especially in trade studies in the preliminary design phase, the achievement of the primary objective requires an onerous iterative calculation regarding fin parameters to optimize its geometrical configuration and dimensions. Thus, a method that can deal with parameter changes easily, as well as calculating temperatures at arbitrary points, would be the best additional tool for thermal designers, even if boundary conditions are simplified at edges.

It is difficult to obtain an exact analytical solution for two-dimensional radiation fins, because the temperature field equation involves a nonlinear radiation heat exchange term. Consequently (except for the nodal network method) temperature distributions of fins have been calculated using the results of theoretical studies on the one-dimensional heat flow across a flat-plate fin uniformly heated on one edge,[1,2] and across a circular fin.[3] These studies can provide only rough estimations for two-dimensional sandwich panel fins because dimensional effects are significant and the heat exchange between inner and outer skins must be considered. Another method uses the implicit finite difference scheme,[4] and is applicable to two-dimensional radiation heat transfer problems of plate-type fins. However, it divides a fin into finite elements in a manner similar to the nodal network method. The method is not simple and cannot easily deal with sandwich panel fins holding an electronic component. Recently, Bobco[5] and Starkovs[6] have simplified the problem by linearizing the radiation term. They developed closed form solutions for a two-dimensional doubler of thin and uniform thickness. This method seems to provide an efficient tool for the optimization of fin design. However, they did not consider sandwich panel-type fins that are the most commonly used type of equipment panel.

In this article, an iterative numerical calculation method, using infinite series expansions of temperature field, is developed for a two-dimensional radiation sandwich panel fin loaded by a uniform heat flux within an arbitrary rectangular footprint region on one surface, and by solar energy on the other. The fin simultaneously radiates to both the ambient satellite environment and the sink environment of space. The

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formulations utilized for the calculation are obtained using the separation of variables technique, and by linearizing the radiation, assuming adiabatic boundary condition at fin edges. The approximation of linearized radiation is considered to be reasonable for applications where the temperature variations or gradients across the fin are not significantly large (as in the equipment panels of communication satellites). The proposed method is developed so that the temperature distribution of each skin can be calculated individually. Following the description of the analytical procedure and zonal formulations to be used in the calculation, the usefulness of the proposed method is confirmed by comparing the calculated temperature distributions with those obtained from nodal network analysis.

Analysis

A schematic representation of the rectangular sandwich panel fin considered here is shown in Fig. 1. The Cartesian coordinates shown in Fig. 1 are commonly applied to both inner and outer skins of the panel. The theoretical formulation of the system is developed on the basis of the following assumptions:

1) The heat transfer, i.e., heat flow, is in a steady state.

2) The fin radiates heat from the outer skin to the sink environment of space that has a constant temperature T_s, and to the ambient satellite environment at a uniform temperature T_a from the inner skin of a honeycomb sandwich panel.

3) The temperature gradient through the thickness of each skin is neglected, because the thickness is very small compared to the other fin dimensions, and the thermal conductivity k_1, k_2 of both skins are high.

4) The heat released from the edge is neglected to simplify the analytical procedure, whereas nodal network analysis is still required to determine design details of fins.

5) The heat flux density q_i is uniformly loaded over the footprint area.

6) Solar energy of flux density q_o is uniformly loaded over the entire outer skin.

Within the framework of these assumptions, a heat balance on differential elements of volume t_1 dx dy and t_2 dx dy at location (x, y) leads to the two-dimensional fin equations

$$\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} = \frac{\sigma \bar{\mathcal{F}}_i (T_1^4 - T_a^4)}{k_1 t_1} + \frac{h(T_1 - T_2)}{k_1 t_1} - \frac{q_i}{k_1 t_1} \tag{1}$$

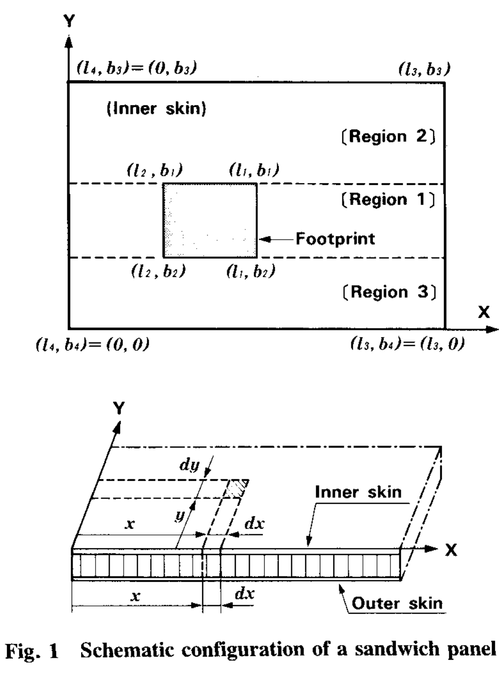


Fig. 1 Schematic configuration of a sandwich panel fin.

$$\frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2} = \frac{\sigma \bar{\mathcal{F}}_o (T_2^4 - T_s^4)}{k_2 t_2} + \frac{h(T_2 - T_1)}{k_2 t_2} - \frac{q_o}{k_2 t_2} \tag{2}$$

$q_i = q_i$ at $l_2 \le x \le l_1$ and $b_2 \le y \le b_1$

$q_i = 0$ at $x < l_2, l_1 < x,$ and $y < b_2, b_1 < y$

where ($\bar{\ }$) denotes mean value with respect to the entire surface area, and each radiation interchange factor is defined as

$$\bar{\mathcal{F}}_i = \varepsilon_i \overline{F_{ia}} \varepsilon_a, \qquad \bar{\mathcal{F}}_o = \varepsilon_o \overline{F_{os}} \varepsilon_s \tag{3}$$

When it is necessary to consider the heat dissipation from the sides of the component, an effective interchange factor $\bar{\mathcal{F}}_e$ that is used in place of $\bar{\mathcal{F}}_i$ in Eq. (1) is represented by the approximate relations

$$A_i \bar{\mathcal{F}}_e = A_i \bar{\mathcal{F}}_i + A_c \bar{\mathcal{F}}_c, \qquad \bar{\mathcal{F}}_i = \varepsilon_i \overline{F_{ia}} \varepsilon_a, \qquad \bar{\mathcal{F}}_c = \varepsilon_c \overline{F_{ca}} \varepsilon_a$$

where A_c and ε_c are the area and the emissivity of the component sides, respectively.

Using mean temperatures T_{1m}, T_{2m} for the inner and the outer skins, Eqs. (1) and (2) are given in linearized radiation and nondimensional coordinate form as

$$\frac{\partial^2 T_1}{\partial \xi^2} + \frac{\partial^2 T_1}{\partial \eta^2} = \frac{(H_1 + h) l_3^2}{k_1 t_1} \left(T_1 - \frac{H_1 T_a + h T_2}{H_1 + h} \right) - \frac{q_i l_3^2}{k_1 t_1} \tag{4}$$

$$\frac{\partial^2 T_2}{\partial \xi^2} + \frac{\partial^2 T_2}{\partial \eta^2} = \frac{(H_2 + h) l_3^2}{k_2 t_2} \left(T_2 - \frac{H_2 T_s + h T_1}{H_2 + h} \right) - \frac{q_o l_3^2}{k_2 t_2} \tag{5}$$

where

$$x = l_3 \xi, \qquad y = l_3 \eta \tag{6}$$

$$H_1 = \sigma \bar{\mathcal{F}}_i (T_{1m}^2 + T_a^2)(T_{1m} + T_a) \tag{7a}$$

$$H_2 = \sigma \bar{\mathcal{F}}_o (T_{2m}^2 + T_s^2)(T_{2m} + T_s) \tag{7b}$$

To calculate the temperature distribution of a sandwich panel fin, Eqs. (1) and (2), or Eqs. (4) and (5) must be solved as simultaneous partial differential equations. However, an alternative calculation method can be proposed to simplify the numerical calculation processes if an iterative calculation of about five iterations is acceptable. The method developed in this article is based on the following idea.

To start the numerical calculation, the mean temperature T_{1m} in Eq. (7a) may be obtained by assuming that the inner skin is a flat-plate fin and radiates energy from one surface directly to space. At this time, the emissivity of the outer surface is assumed to be ε_o. This bulk average temperature T_{1m} corresponds to the value calculated from a simple heat balance that assumes the entire fin radiates at a uniform temperature from both inner and outer surfaces. Using the interim temperature T_{1m}, the mean temperature T_{2m} of the outer skin may be calculated under the energy balance constraint, (i.e., the equilibrium) of the system. After estimating the mean temperatures T_{1m} and T_{2m}, an equivalent sink temperature of space is provisionally determined for the inner skin, and the temperature distribution of the inner skin for the sandwich panel fin can be calculated separately by using them as initial values for Eq. (4). Note that the mean temperatures given as initial values will be different from those obtained after the calculation of the temperature distribution for the inner skin of the panel fin. The discrepancies will be decreased if mean temperatures T_{1m}, T_{2m}, used in the subsequent calculation are modified, considering mean temperatures that are obtained from the previous calculation step. The initial mean temperatures to start the numerical calculation are given by the following relations:

$$A_i q_i + A_o q_o = \sigma[A_i \bar{\mathcal{F}}_i (T_{1m}^4 - T_a^4) + A_o \bar{\mathcal{F}}_o (T_{1m}^4 - T_s^4)] \tag{8}$$

$$A_i h(T_{1m} - T_{2m}) = A_f q_i - A_i H_1 (T_{1m} - T_a) \tag{9a}$$

or

$$\sigma \bar{\mathcal{F}}_o T_{2m}^4 + h T_{2m} = \sigma \bar{\mathcal{F}}_o T_s^4 + h T_{1m} + q_o \tag{9b}$$

Temperature of Inner Skin

The inner skin of the sandwich panel fin does not have direct thermal coupling with the space environment. However, if an equivalent sink temperature of space is decided for the inner skin, the temperature distribution can be calculated assuming the inner skin to be a flat-plate fin. Using the mean temperatures T_{1m}, T_{2m} given by Eqs. (8) and (9), the equivalent sink temperature of space T_{1s} is defined as

$$T_{1s} = (T_{1m}^4 + T_s^4 - T_{2m}^4)^{1/4} \tag{10}$$

Equation (4) is reduced to a tractable form by introducing the newly defined temperature T_{1s}, and written in a nondimensional linearized form as

$$\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} = B_1 \phi_1 - f(\xi) \cdot g(\eta) \tag{11}$$

where

$$\phi_1 = \phi_1(\xi, \eta) = (T_1 - T_{1\infty})(k_1 t_1 / q_i l_3^2) \tag{12}$$

$$T_{1\infty}^4 = \frac{\bar{\mathcal{F}}_i T_a^4 + \bar{\mathcal{F}}_o T_{1s}^4}{\bar{\mathcal{F}}_i + \bar{\mathcal{F}}_o} \tag{13}$$

$$B_1 = \frac{H_{1e} l_3^2}{k_1 t_1} \tag{14}$$

$$H_{1e} = \sigma(\bar{\mathcal{F}}_i + \bar{\mathcal{F}}_o)(T_{1m}^2 + T_{1\infty}^2)(T_{1m} + T_{1\infty}) \tag{15}$$

$$\left.\begin{aligned} f(\xi) &= \begin{cases} 1, \ldots & \xi_2 \le \xi \le \xi_1 \\ 0, \ldots & \xi_4 \le \xi < \xi_2,\ \xi_1 < \xi \le \xi_3 \end{cases} \\ g(\eta) &= \begin{cases} 1, \ldots & \eta_2 \le \eta \le \eta_1 \\ 0, \ldots & \eta_4 \le \eta < \eta_2,\ \eta_1 < \eta \le \eta_3 \end{cases} \end{aligned}\right\} \tag{16}$$

$$C_0 = \frac{\sinh \lambda_n(\eta_2 - \eta_4)\cosh \lambda_n \eta_3 + \sinh \lambda_n(\eta_3 - \eta_1)\cosh \lambda_n \eta_4}{\sinh \lambda_n(\eta_3 - \eta_4)} \tag{22a}$$

$$C_1 = \frac{-\sinh \lambda_n(\eta_3 - \eta_1)\sinh \lambda_n \eta_4 - \sinh \lambda_n(\eta_2 - \eta_4)\sinh \lambda_n \eta_3}{\sinh \lambda_n(\eta_3 - \eta_4)} \tag{22b}$$

Equation (11) can be solved with the separation of variables technique by assuming a separable solution of the form

$$\phi_1(\xi, \eta) = X(\xi) Y(\eta) \tag{17}$$

A solution of $X(\xi)$ may be expressed as a cosine function by applying the boundary condition, i.e.

$$\frac{\partial \phi_1}{\partial \xi} = 0 \quad \text{at} \quad \xi = \xi_3, \xi_4$$

Equation (11) is satisfied if

$$X(\xi) = f(\xi) \tag{18a}$$

$$\frac{d^2 Y}{d\eta^2} - \left[B_1 + \left(\frac{n\pi}{\xi_3}\right)^2\right] Y = -g(\eta) \tag{18b}$$

Equations (18a) and (18b) can be solved by means of Fourier series expansion, and the inhomogeneity with respect to q_i requires a three-zone solution in η. The fin is divided into three zones as shown in Fig. 1, and the zonal solutions are expressed as

$$\phi_{1,m}(\xi, \eta) = X(\xi) \cdot Y_{1,m}(\eta)$$

where $m = 1, 2, 3$ have one-to-one correspondence to the regions $\eta_2 \le \eta \le \eta_1$, $\eta_1 < \eta \le \eta_3$, $\eta_4 \le \eta < \eta_2$. Temperature and temperature gradient matching conditions must be satisfied at zone boundaries in the η direction, i.e.

$$\phi_{1,1}(\xi, \eta_1) = \phi_{1,2}(\xi, \eta_1), \qquad \left.\frac{\partial \phi_{1,1}}{\partial \eta}\right|_{(\xi,\eta_1)} = \left.\frac{\partial \phi_{1,2}}{\partial \eta}\right|_{(\xi,\eta_1)}$$

$$\phi_{1,1}(\xi, \eta_2) = \phi_{1,3}(\xi, \eta_2), \qquad \left.\frac{\partial \phi_{1,1}}{\partial \eta}\right|_{(\xi,\eta_2)} = \left.\frac{\partial \phi_{1,3}}{\partial \eta}\right|_{(\xi,\eta_2)}$$

After some mathematical manipulations, the solution of nondimensional linearized temperature field Eq. (11) is established as

$$\phi_{1,m}(\xi, \eta) = \frac{\xi_1 - \xi_2}{\xi_3} \frac{Y_{0,m}(\eta)}{B_1} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{Y_{n,m}(\eta)}{n \lambda_n^2} \cdot \sin \frac{n\pi(\xi_1 - \xi_2)}{2\xi_3} \cos \frac{n\pi(\xi_1 + \xi_2)}{2\xi_3} \cos \frac{n\pi\xi}{\xi_3} \tag{19}$$

where $\phi_{1,m}(\xi, \eta)$ is the nondimensional parameter representing the temperature distribution, and

$$\lambda_n^2 = B_1 + (n\pi/\xi_3)^2, \qquad (n = 0, 1, 2, \ldots) \tag{20}$$

$$Y_{n,1}(\eta) = 1 - C_0 \cosh \lambda_n \eta - C_1 \sinh \lambda_n \eta \tag{21a}$$

$$Y_{n,2}(\eta) = C_2 \cosh \lambda_n(\eta - \eta_3) \tag{21b}$$

$$Y_{n,3}(\eta) = C_3 \cosh \lambda_n(\eta - \eta_4) \tag{21c}$$

$$C_2 = \frac{\sinh \lambda_n(\eta_1 - \eta_4) - \sinh \lambda_n(\eta_2 - \eta_4)}{\sinh \lambda_n(\eta_3 - \eta_4)} \tag{22c}$$

$$C_3 = \frac{\sinh \lambda_n(\eta_3 - \eta_2) - \sinh \lambda_n(\eta_3 - \eta_1)}{\sinh \lambda_n(\eta_3 - \eta_4)} \tag{22d}$$

Substituting infinite series Eq. (19) into Eq. (12), the formulation to be used in the iterative numerical calculation of the inner skin temperature distribution is given by

$$T_1\left(\frac{x}{l_3}, \frac{y}{l_3}\right) = \phi_{1,m}\left(\frac{x}{l_3}, \frac{y}{l_3}\right) \frac{q_i l_3^2}{k_1 t_1} + T_{1\infty}, \qquad (m = 1, 2, 3) \tag{23}$$

where Eqs. (13) and Eqs. (20–22d) are used to calculate the temperature at any point $(x/l_3, y/l_3)$ on the inner skin.

However, the temperature distribution calculated from Eq. (23) may not be reasonable if the mean temperature of the inner skin (given as the initial value to start the calculation) does not coincide with the mean value obtained after calculation of temperature distribution. That is, the mean temperature of the outer skin, which pairs with the initial mean temperature of the inner skin and is given by the equilibrium Eq. (9), does not approximate the linearized radiation field of the outer skin correctly. Thus, in order to get the most reliable temperature distribution of the inner skin, the iterative calculation using Eq. (23), together with the calculation of the outer skin mean temperature given by Eq. (9a) or (9b), is required until the interim mean temperature of the inner skin coincides with the mean value obtained after calculation of its temperature distribution. It must be noted that the proposed method requires the iterative calculation for the temperature distribution of the inner skin only; the distribution of the outer skin is not always required. This is because the mean temperature of the outer skin, that is utilized to start each iterative calculation, is obtained under the energy balance condition of the entire fin system, and the value becomes a reasonable approximation in parallel with the determination of the most reliable temperature distribution of the inner skin.

The surfaces of the component footprint and the inner skin are assumed to be smooth, and a heat transfer compound possessing high thermal conductivity is applied to minimize the temperature difference between the footprint and the inner skin. In most design cases, however, the temperature of the footprint must be calculated considering the contact resistance between the two surfaces. In the presence of contact resistance, the uniform heat load of the component is represented by using the footprint temperature, $T_e = T_e(x/l_3, y/l_3)$, emissivity of the component surface ε_e, configuration factor between the component and the ambient satellite environment F_{ea}, and the contact conductance D that is the reciprocal of the contact resistance, as

$$q_i = \sigma\varepsilon_e F_{ea}\varepsilon_a(T_e^4 - T_a^4) + D(T_e - T_{1f})$$

where the temperature within footprint region, $T_{1f} = T_{1f}(x/l_3, y/l_3)$, is the inner skin temperature obtained from the calculation mentioned above. The linearized radiation coefficient H_e is then given by

$$H_e = \sigma\varepsilon_e F_{ea}\varepsilon_a(T_{em} + T_a)(T_{em}^2 + T_a^2)$$

where T_{em} denotes the mean value of the temperature T_e. The temperature difference between the footprint and the inner skin, ΔT, can be assumed to be small when the thermal compound bridges the two surfaces. Assuming the mean temperature of T_{1f} as $\bar{T}_{1f}$, the mean temperature of the footprint is represented as

$$T_{em} = \bar{T}_{1f} + \Delta T = \bar{T}_{1f}(1 + \Delta T/\bar{T}_{1f}) \cong \bar{T}_{1f}$$

Thus, H_e can be approximated as

$$H_e \cong \sigma\varepsilon_e F_{ea}\varepsilon_a\bar{T}_{1f}^3\left(1 + \frac{T_a}{\bar{T}_{1f}}\right)\left[1 + \left(\frac{T_a}{\bar{T}_{1f}}\right)^2\right]$$

Substituting H_e into the equation of heat load q_i, the temperature of the footprint is given by

$$T_e\left(\frac{x}{l_3}, \frac{y}{l_3}\right) = \frac{1}{H_e + D}\left[DT_1\left(\frac{x}{l_3}, \frac{y}{l_3}\right) + q_i + H_e T_a\right]$$

where we note that the temperature of the inner skin, $T_1(x/l_3, y/l_3)$, is the value obtained from the proposed iterative calculation method. It is apparent from this equation that T_e equals T_{1f} if we let D equal infinity i.e., zero contact resistance.

Temperature of Outer Skin

The proposed method does not always require the calculation of the outer skin temperature distribution to get the inner skin temperature distribution, as described previously. However, the temperature distribution of the outer skin can be easily calculated by using the proposed method.

The inner skin temperature T_1 at an arbitral location (ξ_j, η_j) is obtained by the iterative calculation using Eqs. (19–23). Thus, the outer skin temperature $T_2(\xi_j, \eta_j)$ at this position can be solved by setting the temperature $T_1(\xi_j, \eta_j)$ in Eq. (5) to a constant value. In order to obtain $T_2(\xi_j, \eta_j)$, the temperature difference between the inner and the outer skins $\Delta T_j(\xi_j, \eta_j)$ is introduced and defined as

$$T_2(\xi_j, \eta_j) = T_{1j}(\xi_j, \eta_j) - \Delta T_j(\xi_j, \eta_j) \tag{24}$$

Thus, the temperature difference $\Delta T_j(\xi_j, \eta_j)$ is expressed as

$$\frac{\partial^2 \Delta T_j}{\partial \xi_j^2} + \frac{\partial^2 \Delta T_j}{\partial \eta_j^2} = B_2(\Delta T_j - T_{2r}) - \frac{q_i l_3^2}{k_1 t_1} \tag{25}$$

$$B_2 = \frac{[(H_2 + h)k_1t_1 + hk_2t_2]l_3^2}{k_1t_1k_2t_2} \tag{26}$$

$$T_{2r} = \frac{(H_2k_1t_1 - H_1k_2t_2)T_{1j} + H_1k_2t_2T_a - (H_2T_s + q_o)k_1t_1}{(H_2 + h)k_1t_1 + hk_2t_2} \tag{27}$$

where the heat dissipation coefficients H_1 and H_2, and the temperature T_{2m} are given by Eqs. (7a), (7b), and (9), respectively. Equation (25) represents the temperature difference between the inner and the outer skins with sufficient accuracy when the temperature distribution of the inner skin is accurately obtained. Therefore, Eq. (25) is recast in nondimensional form by introducing the following relation:

$$\phi_2 = \phi_2(\xi_j, \eta_j) = (\Delta T_j - T_{2r})\frac{k_1t_1}{q_i l_3^2} \tag{28}$$

This relation leads the boundary value problem of Eq. (25) to the expressions

$$\frac{\partial^2 \phi_2}{\partial \xi_j^2} + \frac{\partial^2 \phi_2}{\partial \eta_j^2} = B_2\phi_2 - f(\xi_j)\cdot g(\eta_j) \tag{29}$$

$$\left.\begin{aligned} f(\xi_j) &= \begin{cases} 1, \ldots & \xi_2 \le \xi_j \le \xi_1 \\ 0, \ldots & \xi_4 \le \xi_j < \xi_2,\ \xi_1 < \xi_j \le \xi_3 \end{cases} \\ g(\eta_j) &= \begin{cases} 1, \ldots & \eta_2 \le \eta_j \le \eta_1 \\ 0, \ldots & \eta_4 \le \eta_j < \eta_2,\ \eta_1 < \eta_j \le \eta_3 \end{cases} \end{aligned}\right\} \tag{30}$$

It is found that Eq. (29) has the same form as that derived for the inner skin temperature, as well as that of Eq. (30). Therefore, the solution of linearized partial differential Eq. (29) is given as

$$\phi_{2,m}(\xi_j, \eta_j) = \frac{\xi_1 - \xi_2}{\xi_3}\cdot\frac{Y_{0,m}(\eta_j)}{B_2} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{Y_{n,m}(\eta_j)}{n\mu_n^2}$$
$$\cdot \sin\frac{n\pi(\xi_1 - \xi_2)}{2\xi_3}\cos\frac{n\pi(\xi_1 + \xi_2)}{2\xi_3}\cos\frac{n\pi\xi_j}{\xi_3} \tag{31}$$

$$\mu_n^2 = B_2 + (n\pi/\xi_3)^2, \qquad (n = 0, 1, 2, \ldots) \tag{32}$$

where the expressions for expanding functions $Y_{n,1}$–$Y_{n,3}$ are omitted, since they are obtained by replacing η and λ_n in Eq.

(21) with η_j and μ_n, respectively. Also, the constants C_0–C_3 are obtained by changing λ_n in Eq. (22) to μ_n.

Therefore, using Eqs. (24) and (28) plus infinite series Eq. (31), the formulation to be used in the numerical calculation of the outer skin temperature at the location $(x_j/l_3, y_j/l_3)$, is given by

$$T_2\left(\frac{x_j}{l_3}, \frac{y_j}{l_3}\right) = T_{1j}\left(\frac{x_j}{l_3}, \frac{y_j}{l_3}\right) - \phi_{2,m}\left(\frac{x_j}{l_3}, \frac{y_j}{l_3}\right)\frac{q_i l_3^2}{k_1 t_1} - T_{2r},$$

$$(m = 1, 2, 3) \tag{33}$$

where T_{2r}, given by Eq. (27), represents the equivalent sink temperature at the location $(x_j/l_3, y_j/l_3)$.

Numerical Examples

In order to discuss the validity of the proposed method, three temperature distributions are compared to those calculated by the lumped nodal network analysis method. The examples analyze the temperature distributions of the rectangular sandwich panel fins shown in Table 1. While example 1 has a symmetrical layout, the footprint location in example 2 is arbitrarily chosen to demonstrate the more typical satellite component layout. For the temperature calculation by the nodal method, the fins are divided into equal-sized square elements, and one node is centered on each element.

The temperatures at the positions corresponding to the nodes are also obtained from the calculation using the proposed iterative method. The mean temperature of the inner skin used as the initial value in the proposed method was determined so that the mean value, T_{jm}, obtained after the calculation satisfies the expression

$$AT_m^4 = \sum a_j T_j^4 = AT_{jm}^4, \quad A = \sum a_j, \quad j = 1, 2, 3, \ldots$$

$$|T_m - T_{jm}| \le 0.1$$

where A and T_m denote the surface area and the initial value for the inner skin, and a_j is the area around each position at which the temperature T_j is calculated. The convergence criterion set for temperature T_j was less than 1.5×10^{-7} K. It was found that at least 18 terms of infinite series Eq. (19) for the inner skin, and Eq. (31) for the outer skin, were needed to provide the accuracy mentioned above. The numerical calculations for the following examples were performed using a personal computer equipped with a 32-bit processor.

Example 1

The temperature distribution of this case becomes symmetric with respect to the lines that are parallel to the X and Y axes and pass through the center of the footprint. In this example, two temperature distributions were obtained from both proposed and lumped nodal methods. The temperature variations along the X axis are illustrated to show the differences between the results obtained from both models.

Figures 2a and 2b show the temperatures obtained from the proposed iterative calculation and the node model constructed by 81 equal-sized square elements for the inner and the outer skins, respectively. At the footprint center (0.36, 0.36), the maximum temperature obtained from the proposed method is 1.9°C higher than that calculated by the node model. However, the temperature at (0.04, 0.36), that is close to the fin edge, is 0.6°C lower than the value obtained from the node model. A similar tendency is seen in the temperature distribution of the outer skin, and the differences at positions (0.36, 0.36) and (0.04, 0.36) are 3.7°C and 2.1°C, respectively.

The temperature distributions obtained from the proposed method and the nodal analysis using 324 equal-sized square elements for each skin are shown in Figs. 3a and 3b. The same disagreements seen in Figs. 2a and 2b are repeated in Figs. 3a and 3b. The inner skin temperature at position (0.34, 0.34) obtained from the proposed method is 3.4°C higher than the value calculated by the nodal method, while the temperature at position (0.02, 0.06), near the fin edge, is 1.4°C lower than the value obtained from the nodal method. Moreover, the disagreements between the outer skin temperatures obtained from both methods, show a similar tendency, and the differences are less than 4.9°C.

Using the proposed method enables a thermal designer to calculate a temperature at a position that is specified or chosen arbitrarily. However, the temperature obtained from the nodal analysis does not always correspond to the desired position; the temperatures depend on the definition of finite elements and resultant approximate modeling of the radiation exchanges. The maximum temperature, e.g., whose coordinates are generally unknown, and the temperature at a position specified as a monitoring point are frequently predicted by means of interpolation. The interpolation is achieved by mating the temperatures obtained from the nodal analysis with the nodes. Thus, the maximum temperature at the center

Table 1 Fin parameters for numerical examples

Parameters	Example 1	Example 2
Coordinates of footprint, m (l_1, b_1)	(0.48, 0.48)	(0.44, 0.44)
(l_2, b_2)	(0.24, 0.24)	(0.2, 0.2)
Coordinate of fin, m (l_3, b_3)	(0.72, 0.72)	(0.72, 0.72)
(l_4, b_4)	(0, 0)	(0, 0)
Heat flux at footprint, W/m^2	2,500	2,375
Heat flux at outer skin, W/m^2	0	140
Inner skin thickness; t_1, m	0.002	
Outer skin thickness; t_2, m	0.00025	
Thermal conductivity; k_1, k_2, W/mK	200, 180	
Heat transfer rate between skins, W/m^{2}K	40.0	
Environment temperature; T_a, T_s, K	265, 3	
Emissivity; ε_i, ε_a, ε_o, ε_s	0.8, 1.0, 0.8, 1.0	
Configuration factor; F_{ia}, F_{os}	1.0, 1.0	

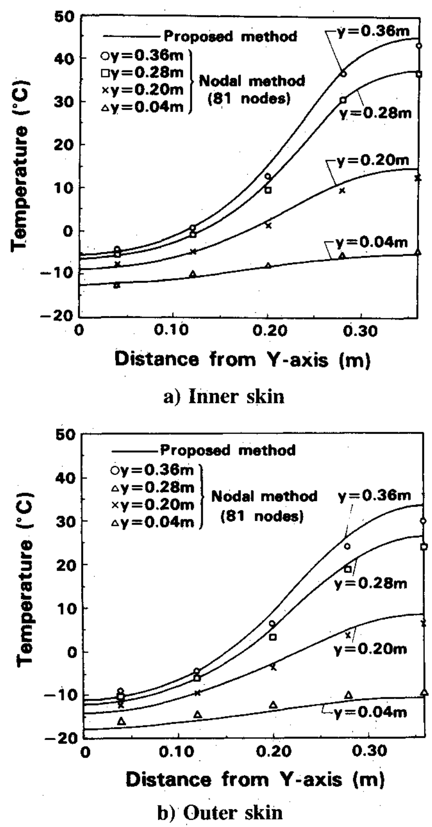
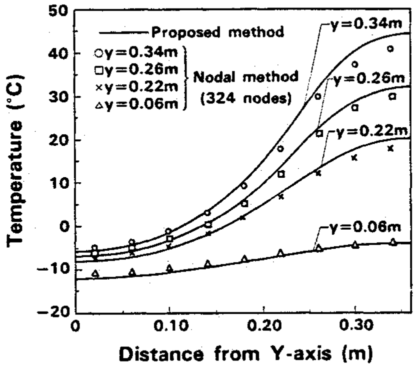
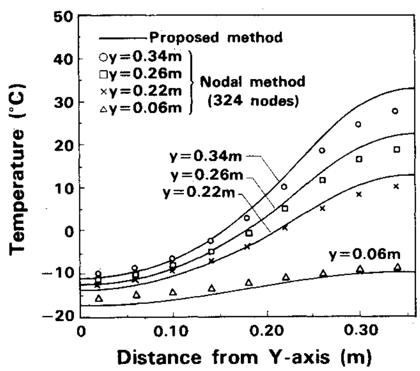


Fig. 2 Comparison of temperatures obtained from proposed method and nodal analysis of example 1.



a) Inner skin



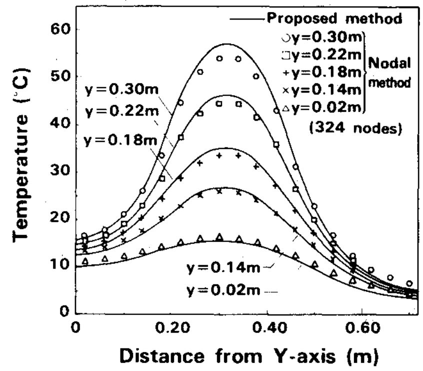
b) Outer skin

Fig. 3 Comparison of temperatures obtained from proposed method and nodal analysis of example 1.

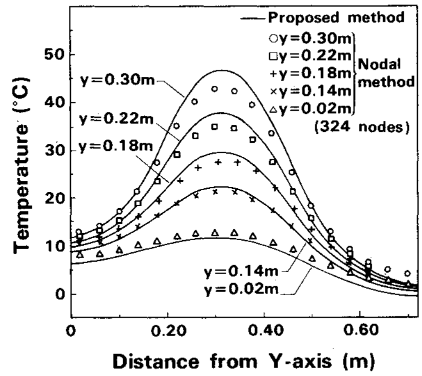
of the footprint was obtained by this interpolation method, because no element enclosing the center was used in the inner skin nodal model—even with 324 nodes. The maximum temperature of 42.5°C was obtained from the interpolation using a biquadratic polynomial expression. This maximum temperature was 1.5°C lower than that obtained from the nodal model that was represented by 81 nodes. Also the temperature at position (0.28, 0.36) near the footprint boundary was interpolated, since the temperature at that position was not obtained from the inner skin model of 324 nodes. The temperature of 34.0°C was yielded after interpolation using the same order polynomial, and was 2.6°C lower than that obtained from the nodal model represented by 81 nodes. It can be considered that these results illuminate uncertainty of the temperatures predicted by numerical calculations.

Example 2

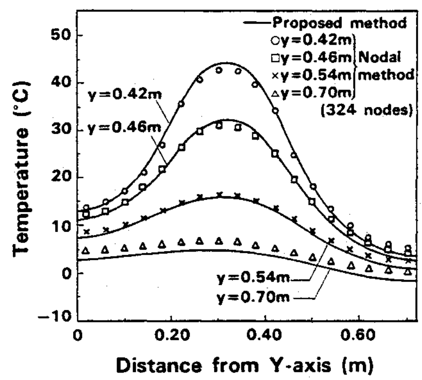
The temperature distributions calculated by using the proposed method and the nodal method are shown in Figs. 4a–4d. The inner and the outer skins of the fin were divided into 324 equal-sized square elements for nodal analysis. The temperature obtained using the proposed method is 2.5°C higher than that by the node model at position (0.3, 0.3) on the inner skin, that is close to the maximum temperature position. However, the temperatures along the coordinate value of $x = 0.7$ are 1.7°C lower than those obtained from the node model. As for the temperatures of the outer skin, the disagreement is 4.0°C at position (0.3, 0.3) and 1.5–2.7°C at positions along $x = 0.7$, which are near the fin periphery. As observed in example 1, the disagreements near the fin edges mainly resulted from the definitions of the isothermal elements in the node models that have no nodes at the edges. Furthermore, it was also considered that such node definition had a smoothing effect on the inner and outer skins temperature distributions in both examples. Consequently, the calculation method yielded the temperature disagreements of -1.8–2.5°C for the inner skin and -4.0–4.0°C for the outer skin.



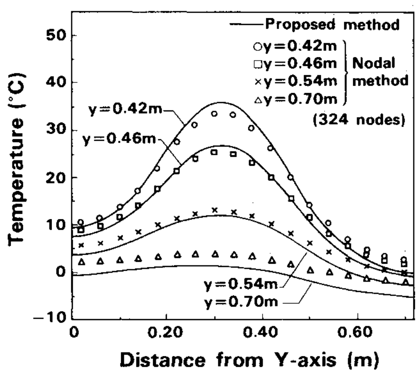
a) Inner skin



b) Outer skin



c) Inner skin



d) Outer skin

Fig. 4 Comparison of temperatures obtained from proposed method and nodal analysis of example 2.

Discussion

Based on the results obtained from examples 1 and 2, it is considered that the maximum temperatures obtained from the proposed method may be slightly higher than those given by nodal analysis; the heat dissipation coefficients are calculated from the mean temperature over the entire surface area, and are smaller than those at the elements including the maximum temperature points. In example 1, an increase in the inner skin thickness from 2.0 to 2.09 mm is needed to assure the maximum temperature of 43.5°C that was obtained from the nodal method. However, the temperature distribution obtained from the proposed method enables us to reduce the doubler's length and width by about 2 cm each; the temperatures near the periphery are lower than those obtained from the nodal method. Therefore, the difference of the maximum temperatures may not be a significant disadvantage insofar as the maximum temperature difference of a few degrees yields a more conservative design with regard to fin thickness.

A concept of temperature prediction uncertainty is basically essential for assessing the calculation results regardless of which method is used, because the temperature is treated as constant for radiation on a differential element and treated as variable only for the conduction through the element. Thus, for the industrial applications, it may be reasonable to consider that the uncertainty in the nodal method is within ±5°C,[7] because of the variety of uncertainties induced by the approximate definition of finite and isothermal elements, and the approximate modeling of radiative and conductive exchange between isothermal elements. The difference in the maximum temperature obtained from 81 × 2 and 324 × 2 element models in example 1 can be considered to be one proof of the empirical fact mentioned above. The temperature uncertainty introduced into the nodal analysis method can be also adopted in the proposed calculation method due to the approximate modeling of the radiation exchange. Thus, the temperature difference of ±5°C between the results obtained from the proposed and the nodal methods is treated as an acceptable disagreement in this paper. The numerical results of examples 1 and 2 revealed that the disagreements of the temperature distributions obtained from both methods were less than ±5°C, and satisfied the criterion. Therefore, it can be concluded that the proposed calculation method is useful for trade studies of sandwich panel fins where the temperture differences between the inner and the outer skins are not excessive (as is true in examples 1 and 2).

The proposed calculation method yields the most reliable result when the mean temperature utilized as an initial value for the inner skin coincides with the mean value obtained after the calculation of the temperature distribution. To confirm this, the influences of the mean temperatures used as initial values on the resultant temperature distributions were investigated using examples 1 and 2. The mean temperatures finally obtained varied by ±2°C and used as the initial values. The results showed that the ratios of the maximum temperature change, and the minimum temperature change to the mean temperature change, were −0.35 and −0.30 in both examples 1 and 2, respectively. This result means that the discrepancy of the inner skin mean temperature between the initial value, and the resulting value, must be decreased to less than ±2.8°C by performing iterative calculations if the maximum temperature is to be obtained with an accuracy of ±1°C. Note, however, that this accuracy does not always require many iterative calculations. For instance, the most reliable temperature distributions for examples 1 and 2 were obtained in less than five iterations.

Concluding Remarks

An iterative calculation method using infinite series expansions has been developed to determine the steady-state temperature distribution across a two-dimensional rectangular sandwich panel fin with thin skins. The fin receives a heat flux from within a rectangular footprint region on the inner skin and solar energy loads on the outer skin. Energy is dissipated to both environments by linearized radiation. The formulations to be used in the calculation are derived by taking into account the configuration, simplified boundary condition, and the other design parameters, to approximate a satellite equipment panel holding electronic components.

Numerical results revealed that the uncertainty of the temperature prediction obtained from the proposed method, was comparable to that obtained from the lumped nodal network analysis method (when the temperature difference between the inner and the outer skins was not excessive). It was also shown that the proposed method could individually calculate the temperture of each sandwich panel fin skin. Therefore, it can be concluded that the proposed calculation method is useful in evaluating design parameters of sandwich panel fins, especially in trade studies in preliminary design phase. However, design details for fins must be determined using results obtained from lumped nodal network analysis, since the proposed method is based on simplified boundary conditions.

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